Self-Iterating Soft Equalizer

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***Abstract*—We design a self-iterating soft equalizer (SISE) consisting of several suboptimal equalizers that are weak individually but, when working together, show good performance. In order to process correlated soft information through serially concatenated modules, a sophisticated method to generate the extrinsic information is proposed that suppresses the correlation effect between the soft information of suboptimal equalizers. This algorithm performs well in turbo equalization system as well as the classic uncoded equalization system. The performance advantages are validated with bit-error-rate (BER) simulations and extrinsic information transfer (EXIT) chart analysis.**

I. Introduction

It has been widely recognized that the turbo equalization method developed in [1] is very effective in removing intersymbol interference (ISI) and improving bit error rate (BER) performance even at low signal to noise ratios (SNRs). The original turbo equalization system basically consists of two constituent modules, a soft-in soft-out (SISO) equalizer and a SISO error-correction decoder, separated by an interleaver with the two modules exchanging soft information in an iterative fashion.

The optimal equalizer for minimizing the BER in the turbo equalization system is based on the well-known Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [2], which performs maximum *a posteriori* probability (MAP) estimation for each bit. However, an issue with the BCJR algorithm is the exponentially growing computational complexity as a function of the channel length and the symbol alphabet set size.

Numerous suboptimal but low complexity turbo equalization schemes have been proposed to mitigate the high complexity of the BCJR-based equalizer [3], [4], [5], [6], [7], [8]. These suboptimal equalizers can be classified into: SISO linear equalizer (LE), SISO decision feedback equalizer (DFE), or SISO soft-feedback equalizer (SFE). Recently, it has been also shown [8], [9], [10], [11], [12] that employing two DFEs (or SFEs) running in opposite directions and combining their extrinsic information is very effective in turbo equalization systems as well as in classic equalization systems. Moreover, unlike the LE and DFE, this bidirectional DFE (BiDFE) algorithm does not suffer from performance degradation even when the filter taps are constrained to be time-invariant [8]. The BiDFE algorithm can be considered as a "parallel" concatenated scheme with two suboptimal DFEs producing correlated but different extrinsic information.

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In this paper, we propose a self-iterating soft equalizer (SISE) consisting of several weak suboptimal equalizers which are serially concatenated. We try to find a way by which one suboptimal equalizer can profit from other suboptimal equalizers which are "serially" concatenated and produce somewhat correlated but different enough extrinsic information. The rationale behind this particular equalizer structure is that the suboptimal equalizers such as LE, DFE, and BiDFE have their own advantages and disadvantages, and one should benefit from the presence of the other equalizers. For example, the LE does not have the error propagation problem which the DFE suffers from, whereas the DFE shows better performance than the LE when feedback decisions are correct; and BiDFE shows good performance even with time-invariant filters, although its complexity is roughly double the complexity of the DFE, when similar filter components are used.

The remainder of the paper is organized as follows. In Section II, a brief statement of the system model is given. In Section III, we show a proper way to adopt the extrinsic information from the other modules where the information between the modules could be highly correlated and then propose self-iterating soft equalizer design for uncoded systems. We also provide the turbo equalization algorithm based on the SISE in Section IV. In Section V, numerical results and analysis are given. Finally, we draw conclusions in Section VI.

II. System Model

Given the transmitted sequence of coded bits $\{x_k\}$, the ISI channel output at time n is

$$r_n = \sum_{k=0}^{L_h-1} h_k x_{n-k} + w_n \tag{1}$$

where w_n is additive white Gaussian noise (AWGN) with variance N_0 and $\{h_k\}$ is the channel impulse response with length L_h. In this paper, it is assumed that the transmitted symbol is a binary input with the average power equal to 1, i.e., $x_n \in \{\pm 1\}$ and $P_X \triangleq \mathrm{E}(x_n^2) = 1$, and the ISI channel coefficients and noise samples are real valued.

In turbo equalization, the *a priori* log-likelihood ratio (LLR) of x_n to the equalizer is defined as

$$L_a(x_n) \triangleq \ln \frac{\Pr(x_n = +1)}{\Pr(x_n = -1)}$$

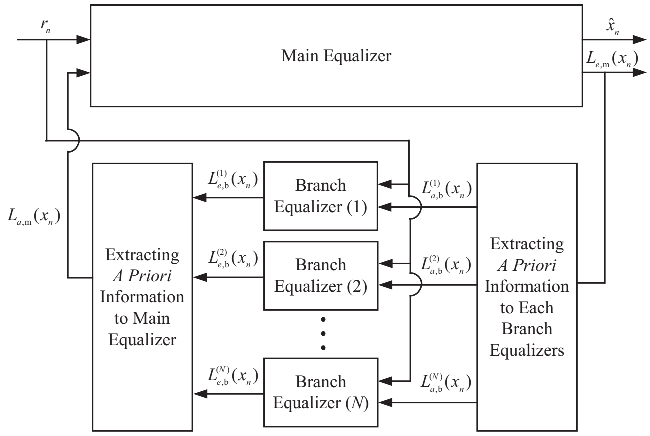

Fig. 1: Self-Iterating Soft Equalizer.

where the probabilities in the expression are in reality just estimates.

Based on these *a priori* LLR values for the symbols, the equalizer generates its own extrinsic information, which will in turn be passed to the decoder. One way of generating the extrinsic LLR $L_e(x_n)$ is to set $L_a(x_n) = 0$ while computing the *a posteriori* LLR $L(x_n)$ [3]. Let y_n be the equalizer filter output sequence corresponding to the observation sequence r_n applied at the input by setting $L_a(x_n) = 0$ during the computation of y_n. Then, the equalizer's extrinsic information is directly related to the equalizer output y_n as:

$$\begin{aligned}
L_e(x_n) &\triangleq L(x_n)|_{L_a(x_n)=0} \\
&= \ln \frac{\Pr(x_n = +1 \mid y_n)}{\Pr(x_n = -1 \mid y_n)}\bigg|_{L_a(x_n)=0} \\
&= \ln \frac{\Pr(y_n \mid x_n = +1)\Pr(x_n = +1)}{\Pr(y_n \mid x_n = -1)\Pr(x_n = -1)}\bigg|_{L_a(x_n)=0} \\
&= \ln \frac{\Pr(y_n \mid x_n = +1)}{\Pr(y_n \mid x_n = -1)}. \qquad (2)
\end{aligned}$$

III. Self-Iterating Soft Equalizer Algorithm

In this section, we discuss a self-iterating soft equalizer (SISE) algorithm. Basically, the channel equalizer is a SISO equalizer which consists of one main suboptimal SISO equalizer and N branch suboptimal SISO equalizers. The proposed scheme of SISE is shown in Fig. 1.

The main idea of this algorithm is that the received data sequence is equalized by the main equalizer and its extrinsic information is passed to the branch equalizers as their *a priori* information. The extrinsic information of the branch equalizers are also passed back to the main equalizer as the *a priori* information for the next equalization process. Note that since this equalization algorithm can perform iteratively without the decoder (hence the name "self-iterating soft equalizer"), it can be used in uncoded systems.

However, unlike the extrinsic information between the decoder and the equalizer in usual turbo equalization, the extrinsic information between the main equalizer and the branch equalizers have correlation because no interleaving techniques can be used and their equalization processes are all based on the common received data sequence. Therefore, the main equalizer cannot easily adopt the extrinsic information from the branch equalizers [13], [14]. In this section, we show a proper way to adopt the extrinsic information from the branch equalizers when the main equalizer and the branch equalizers are highly correlated.

A. Generation of Uncorrelated A Priori Information

First, let us assume that there is one main equalizer and one branch equalizer in an uncoded system. This assumption is not necessary for our development and we will later extend the proposed algorithm based on this simple case. Furthermore, we also assume that the main equalizer passes the extrinsic LLR, $L_{e,\mathrm{m}}(x)$, to the branch equalizer as its *a priori* LLR, $L_{a,\mathrm{b}}(x)$, and the branch equalizer produces the extrinsic LLR, $L_{e,\mathrm{b}}(x)$ with the given $L_{a,\mathrm{b}}(x)$.

Since the *a priori* LLR (or extrinsic LLR) can be modeled as coming out of an equivalent AWGN channel [15], let us consider two unbiased equalizer outputs, which are corrupted by AWGN, corresponding to the transmitted symbol x:

$$y_\mathrm{m} = x + u_\mathrm{m}, \quad y_\mathrm{b} = x + u_\mathrm{b}$$

where the subscript 'm' and 'b' indicate the main equalizer and the branch equalizer, respectively. Here, time index n is dropped for notational simplicity, but the process remains identical as n progresses. The noise terms u_m and u_b are assumed to be zero mean Gaussian random variables which are independent of the transmitted data x but correlated with each other with correlation coefficient ρ.

Since the extrinsic LLR of the main equalizer is used for the estimated *a priori* LLR to the branch equalizer, the *a priori* LLR of the branch equalizer can be written as

$$\begin{aligned}
L_{a,\mathrm{b}}(x) &= \ln \frac{\Pr(x = +1)}{\Pr(x = -1)} \\
&= L_{e,\mathrm{m}}(x) \\
&= \ln \frac{\Pr(y_\mathrm{m}|x = +1)}{\Pr(y_\mathrm{m}|x = -1)}. \qquad (3)
\end{aligned}$$

Therefore, it can be considered that the *estimated* events $\{x = +1\}$ and $\{x = -1\}$ are identical to the events $\{y_\mathrm{m}|x = +1\}$ and $\{y_\mathrm{m}|x = -1\}$ respectively. Then, the *a posteriori* LLR of the branch equalizer can be written as

$$\begin{aligned}
L_\mathrm{b}(x) &= \ln \frac{\Pr(x = +1|y_\mathrm{b})}{\Pr(x = -1|y_\mathrm{b})} \\
&= \ln \frac{\Pr(y_\mathrm{b}, \{x = +1\})}{\Pr(y_\mathrm{b}, \{x = -1\})} \\
&= \ln \frac{\Pr(y_\mathrm{b}, \{y_\mathrm{m}|x = +1\})}{\Pr(y_\mathrm{b}, \{y_\mathrm{m}|x = -1\})} \\
&= \ln \frac{\Pr(y_\mathrm{b}, y_\mathrm{m}|x = +1)}{\Pr(y_\mathrm{b}, y_\mathrm{m}|x = -1)} \qquad (4) \\
&= \frac{\left(N_\mathrm{b} - \rho\sqrt{N_\mathrm{m}N_\mathrm{b}}\right)}{(1-\rho^2)\,N_\mathrm{b}} L_{e,\mathrm{m}}(x) \\
&\quad + \frac{\left(N_\mathrm{m} - \rho\sqrt{N_\mathrm{m}N_\mathrm{b}}\right)}{(1-\rho^2)\,N_\mathrm{m}} L_{e,\mathrm{b}}(x) \qquad (5)
\end{aligned}$$

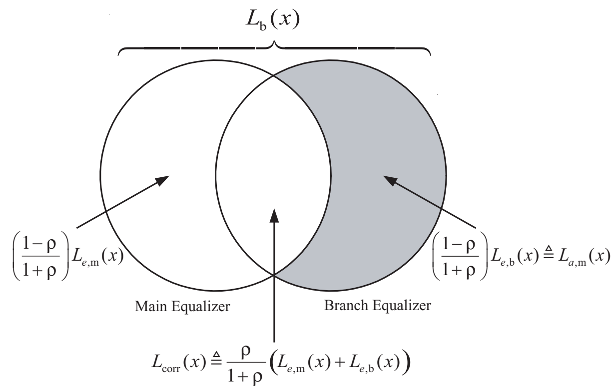

Fig. 2: Venn Diagram of Branch Equalizer's *A Posteriori* Information.

where $N_\text{m} \triangleq \text{Var}(u_\text{m})$ and $N_\text{b} \triangleq \text{Var}(u_\text{b})$. The equality of (4) holds because y_m and y_b should contain the same transmitted symbol x. The detail derivation of (5) is given in [8]. Moreover, due to the sensitivity of $L_\text{b}(x)$ to the estimation error of ρ, [8], we further assume that the variance of u_m and u_b are the same, i.e., $N_\text{m} = N_\text{b}$. Then,

$$L_\text{b}(x) = \frac{1}{1+\rho}\Big(L_{e,\text{m}}(x) + L_{e,\text{b}}(x)\Big). \tag{6}$$

Now, we discuss how to extract the extrinsic information from this *a posteriori* LLR of the branch equalizer as the *a priori* LLR of the main equalizer. Notice that if the noise u_m and u_b are uncorrelated, i.e., $\rho = 0$, the *a posteriori* LLR of the branch equalizer is given as $L_\text{b}(x) = L_{e,\text{m}}(x) + L_{e,\text{b}}(x)$ and the *a priori* LLR to the main equalizer is naturally defined as $L_{a,\text{m}}(x) = L_\text{b}(x)|_{L_{e,\text{m}}(x)=0} = L_{e,\text{b}}(x)$. However, due to the correlation between noise, $L_\text{b}(x)$ is reduced by the factor of $1/(1+\rho)$. Therefore, the correlated information between the extrinsic information of the main equalizer and the extrinsic information of the branch equalizer can be defined as

$$\begin{aligned} L_\text{corr}(x) &\triangleq L_\text{b}(x)|_{\rho=0} - L_\text{b}(x) \\ &= \frac{\rho}{1+\rho}\Big(L_{e,\text{m}}(x) + L_{e,\text{b}}(x)\Big). \end{aligned} \tag{7}$$

We can view the relationship between $L_\text{b}(x)|_{\rho=0}$, $L_\text{b}(x)$, and $L_\text{corr}(x)$ in this sense: Since the correlated information is added twice in $L_\text{b}(x)|_{\rho=0}$, it is necessary to subtract the correlated information, $L_\text{corr}(x)$, from $L_\text{b}(x)|_{\rho=0}$ to get $L_\text{b}(x)$.

Then, in an effort to avoid exaggerating the effect of soft information, the *a priori* LLR of the main equalizer from the branch equalizer can be defined as

$$\begin{aligned} L_{a,\text{m}}(x) &\triangleq \Big(L_\text{b}(x) - L_\text{corr}(x)\Big)\Big|_{L_{e,\text{m}}(x)=0} \\ &= \left(\frac{1-\rho}{1+\rho}\right) L_{e,\text{b}}(x). \end{aligned} \tag{8}$$

The reason why we define $L_{a,\text{m}}(x)$ in this way is illustrated in Fig. 2. As seen in the Venn Diagram of $L_\text{b}(x)$, only the information in colored area should be fed back to the main equalizer since this information is extrinsic to the main equalizer. Notice that if two noise samples are uncorrelated such as the extrinsic information between the decoder and the equalizer, we can fully utilize the extrinsic LLR of the branch equalizer as the *a priori* LLR of the main equalizer, i.e., $L_{a,\text{m}}(x) = L_{e,\text{b}}(x)$. On the other hand, if the noise correlation coefficient is 1, the extrinsic information of the branch equalizer is redundant to the main equalizer, i.e., $L_{a,\text{m}}(x) = 0$.

Note that this process is also valid in opposite direction, i.e., when the *a priori* LLR of the branch equalizer is extracted from the *a posteriori* LLR of the main equalizer.

B. Estimation of Noise Correlation Coefficient

Unfortunately, it is difficult to compute the correlation coefficient between u_m and u_b (or $L_{e,\text{m}}(x)$ and $L_{e,\text{b}}(x)$) analytically. However, assuming that the noise is stationary, the correlation coefficient can be estimated through time-averaging with $L_{e,\text{m}}(x)$ and $L_{e,\text{b}}(x)$ over some reasonably large finite window. In other words, the correlation coefficient can be computed as (10) where, due to the symmetricity of x_n, m_m and m_b are the mean values of $L_{e,\text{m}}(x_n)$ and $L_{e,\text{b}}(x_n)$ when $x_n = +1$ respectively, i.e., $m_\text{m} = \text{E}(L_{e,\text{m}}(x_n)|x_n = +1)$ and $m_\text{b} = \text{E}(L_{e,\text{b}}(x_n)|x_n = +1)$. We can also estimate them through time-averaging:

$$\begin{aligned} \hat{m}_\text{m} &= \overline{(L_{e,\text{m}}(x_n)|L_\text{m}(x_n) \geq 0)} \\ \hat{m}_\text{b} &= \overline{(L_{e,\text{b}}(x_n)|L_\text{b}(x_n) \geq 0)}. \end{aligned}$$

where $\overline{t_n}$ means the time-average of t_n. Note that the hard decisions for the transmitted symbols in the main equalizer and the branch equalizer might be different; in estimating the correlation coefficient, we only consider those *a posteriori* LLR samples for which $\text{sign}\left[L_\text{m}(x_n)\right]$ and $\text{sign}\left[L_\text{b}(x_n)\right]$ are identical. Also, notice that $L_{e,\text{m}}(x_n)$ in (10) can be replaced by $L_{a,\text{b}}(x_n)$.

C. Extension of Simple Case

In order to show the extended SISE algorithm inductively, now, let us assume that there is one main equalizer and two branch equalizers. Then, the *a priori* LLR to the main equalizer from each branch equalizer can be defined as

$$\begin{aligned} L_{a,\text{m}}^{(1)}(x) &= \left(\frac{1-\rho^{(1)}}{1+\rho^{(1)}}\right) L_{e,\text{b}}^{(1)}(x) \\ L_{a,\text{m}}^{(2)}(x) &= \left(\frac{1-\rho^{(2)}}{1+\rho^{(2)}}\right) L_{e,\text{b}}^{(2)}(x) \end{aligned}$$

where superscript (i) points to a specific branch equalizer and $\rho^{(i)}$ is the correlation coefficient between $L_{e,\text{b}}^{(i)}(x)$ and $L_{a,\text{b}}^{(i)}(x)$. Again, since the *a priori* LLR (or extrinsic LLR) can be modeled as an equivalent AWGN channel output, under the assumption that the noise variances are the same, the whitened and combined *a priori* information to the main equalizer, [8], is given by

$$L_{a,\text{m}}(x) = \frac{1}{1+\xi}\Big(L_{a,\text{m}}^{(1)}(x) + L_{a,\text{m}}^{(2)}(x)\Big) \tag{11}$$

$$\hat{\rho} = \frac{\sum \left\{ \left(L_{e,\mathrm{m}}(x_n) - \text{sign}\left[L_{\mathrm{m}}(x_n) \right] m_{\mathrm{m}} \right) \left(L_{e,\mathrm{b}}(x_n) - \text{sign}\left[L_{\mathrm{b}}(x_n) \right] m_{\mathrm{b}} \right) \right\}}{\sqrt{\sum \left(L_{e,\mathrm{m}}(x_n) - \text{sign}\left[L_{\mathrm{m}}(x_n) \right] m_{\mathrm{m}} \right)^2} \sqrt{\sum \left(L_{e,\mathrm{b}}(x_n) - \text{sign}\left[L_{\mathrm{b}}(x_n) \right] m_{\mathrm{b}} \right)^2}} \tag{10}$$

where ξ is the noise correlation coefficient between $L^{(1)}_{a,\mathrm{m}}(x)$ and $L^{(2)}_{a,\mathrm{m}}(x)$ and it can be also estimated through time-averaging using a similar equation to (10). Based on this property, we can further extend the SISE algorithm to one consisting of N branch equalizers.

D. SISE Algorithm

Finally, the proposed SISE algorithm for an uncoded system can be summarized as follows:

- Initialize the *a priori* LLR of the main equalizer, i.e., $L_{a,\mathrm{m}}(x_n) = 0$ and $L^{(i)}_{a,\mathrm{m}}(x_n) = 0$ for all time index n and branch index i.
- For the specified number of self-iterations,
  1) Generate the extrinsic LLR of the main equalizer, $L_{e,\mathrm{m}}(x_n)$, with the *a priori* LLR $L_{a,\mathrm{m}}(x_n)$ for all n.
  2) Compute the noise correlation coefficients, $\rho^{(i)}_{\mathrm{m}}$, between $L_{e,\mathrm{m}}(x_n)$ and $L^{(i)}_{a,\mathrm{m}}(x_n)$ and set $L^{(i)}_{a,\mathrm{b}}(x_n) = (1-\rho^{(i)}_{\mathrm{m}})/(1+\rho^{(i)}_{\mathrm{m}}) \cdot L_{e,\mathrm{m}}(x_n)$ for all i.
  3) Generate the extrinsic LLR of each branch equalizer, $L^{(i)}_{e,\mathrm{b}}(x_n)$, with the given *a priori* LLR $L^{(i)}_{a,\mathrm{b}}(x_n)$ for all i.
  4) Compute the noise correlation coefficient, $\rho^{(i)}_{\mathrm{b}}$, between $L^{(i)}_{e,\mathrm{b}}(x_n)$ and $L^{(i)}_{a,\mathrm{b}}(x_n)$ and set $L^{(i)}_{a,\mathrm{m}}(x_n) = (1-\rho^{(i)}_{\mathrm{b}})/(1+\rho^{(i)}_{\mathrm{b}}) \cdot L^{(i)}_{e,\mathrm{b}}(x_n)$ for all i.
  5) Generate the *a priori* LLR of the main equalizer, $L_{a,\mathrm{m}}(x_n)$, with $L^{(i)}_{a,\mathrm{m}}(x_n)$ via the extended equation of (11).

IV. Iterative Self-Iterating Soft Equalizer Algorithm

In this section, we propose turbo equalization based on the previously developed SISE algorithm in conjunction with the decoder. The structure of the turbo equalization scheme based on SISE is shown in Fig. 3. Various iterative equalization algorithms are possible in this structure, but two main algorithms are proposed in this paper.

A. SISE 1 Algorithm

The first algorithm passes the uncorrelated extrinsic LLR of the main equalizer to the branch equalizers and to the decoder in turn.

- Initialize the *a priori* LLR from the decoder and the branch equalizers, i.e., $L_a(x_n) = 0$, $L_{a,\mathrm{m}}(x_n) = 0$, and $L^{(i)}_{a,\mathrm{m}}(x_n) = 0$ for all time index n and branch index i.
- For the specified number of outer iterations,
  1) Generate the extrinsic LLR of the main equalizer, $L_{e,\mathrm{m}}(x_n)$, with the *a priori* LLR from the decoder, $L_a(x_n)$, for all n.

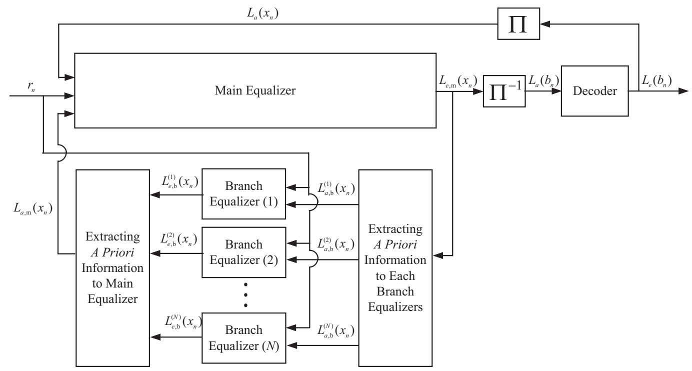

Fig. 3: Turbo Self-Iterating Soft Equalization Scheme.

  2) Take the same steps of $2) - 5)$ in III-D.
  3) Reproduce the extrinsic LLR of the main equalizer, $L_{e,\mathrm{m}}(x_n)$, with the combined *a priori* LLR from the decoder and the branch equalizers, $L_a(x_n) + L_{a,\mathrm{m}}(x_n)$.
  4) Pass $L_{e,\mathrm{m}}(x_n)$ back to the decoder.

B. SISE 2 Algorithm

Due to the separate self-iteration step of the SISE 1 algorithm, it has long processing latency issue; therefore, the second algorithm (SISE 2) is also proposed. Different from the first algorithm, SISE 2 passes the uncorrelated extrinsic information of the main equalizer to the branch equalizers and to the decoder simultaneously and the self-iteration is performed during the outer turbo iteration is processed.

- Initialize the *a priori* LLR from the decoder and the branch equalizers, i.e., $L_a(x_n) = 0$, $L_{a,\mathrm{m}}(x_n) = 0$, and $L^{(i)}_{a,\mathrm{m}}(x_n) = 0$ for all time index n and branch index i.
- For the specified number of outer iterations,
  1) Generate the extrinsic LLR of the main equalizer, $L_{e,\mathrm{m}}(x_n)$, with the combined *a priori* LLR from the decoder and the branch equalizer, $L_a(x_n) + L_{a,\mathrm{m}}(x_n)$, for all n.
  2) Pass $L_{e,\mathrm{m}}(x_n)$ to the decoder.
  3) While the decoder computes its own extrinsic LLR and passes it as $L_a(x_n)$ to the main equalizer,
     – Take the same steps of $2) - 5)$ in III-D.

C. Comparison of Complexity and Latency

Let the computational complexity of the main equalizer, the branch equalizers, and the decoder be C_M, C_B, and C_D, respectively. For each outer iteration performed, the amount of computation for the conventional turbo equalization is $C_M + C_D$, whereas it is $2C_M + C_B + C_D$ and $C_M + C_B + C_D$ for SISE 1 and SISE 2, respectively. Moreover, assuming the processing time for the main equalizer, the parallel branch equalizers, and the decoder is all equal to T, the total processing time for each outer iteration is $2T$, $4T$, and $2T$ for the conventional system, SISE 1, and SISE 2, respectively. While SISE 2 requires higher complexity (by C_B) than existing turbo equalizers when C_M is fixed in both cases, it will be shown that SISE 2 often enables substantial error rate reduction in channel conditions where the existing turbo equalizer cannot provide any performance improvement regardless of how large C_M is allowed to grow.

V. SIMULATION RESULTS

In this section, simulation results of the proposed SISE equalization scheme for both uncoded system and coded system are presented. The transmitted symbols are modulated by binary phase-shift keying (BPSK) method so that $x_n \in \{\pm 1\}$ with 2^{11} message bits. We also assume that the noise is AWGN, and the noise variance and the channel information are perfectly known to the receiver. In the legend, the notations "TV-" and "TI-" mean equalizers with time-varying filters and time-invariant filters respectively, as introduced in [3]. For instance, "TV-BiDFE" in the legend indicates the BiDFE algorithm with time-varying filters. Moreover, the label "Ideal" indicates the performance of an equalizer with perfect *a priori* information, i.e., $L_a(x_n) = \pm\infty$.

A. Uncoded System

The ISI channel $\mathbf{h_1} = (1/\sqrt{6})[1 \quad 2 \quad 1]^T$ is used for the uncoded system. The legend "SISE" indicates the self-iterating soft equalizer described in Section III-D. Specifically, the SISO TI-BiDFE algorithm of [8] is adopted as the main equalizer which takes into account the possibility that error propagation i_n is not zero by tracking the conditional extrinsic LLR: $L_e(x_n|i_n \neq 0) = 2p_{\{n,0\}}\,(y_n - \mathrm{E}(i_n|i_n \neq 0))/\mathrm{Var}(v_n)$. The detail description is given in [8]. Moreover, when decisions are made by BiDFE, the arbitration criterion of [12] with window size 15 is also adopted; the symbol sequence is decided among the estimated sequence of two DFEs based on which candidate shows the smaller mean-squared-error in a window around the symbol of interest. Finally, the SISO TI-LE of [3] is used for the branch equalizer and 2 self-iterations are applied. The "BAD", algorithm of [12] not employing the branch equalizer LE, and the "MAP", the optimal equalizer implemented via the BCJR algorithm of [2], are simulated for performance comparison purpose. Each DFE in BiDFE consists of 13 feedforward taps and 2 feedback taps while the LE uses 15 taps for $\mathbf{h_1}$. As the Fig. 4 shows, the proposed SISE algorithm shows the superior performance to the BAD method of [12] and approaches the performance of "Ideal BAD".

B. Coded System

In this subsection, simulation results of the iterative SISE equalization scheme are also presented. The transmitted symbols are encoded with a recursive rate-1/2 convolutional code encoder with parity generator $(1 + D^2)/(1 + D + D^2)$. The impulse response of the ISI

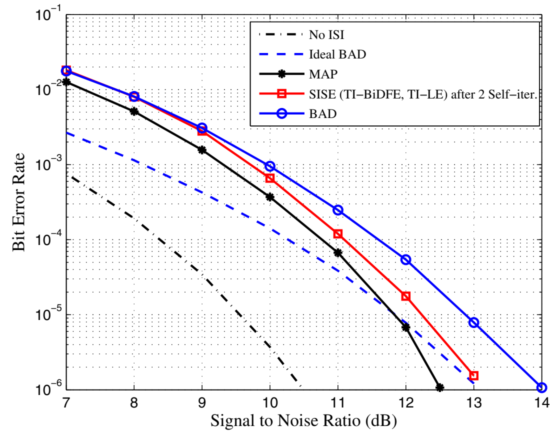

Fig. 4: BER Curves on the Channel $\mathbf{h_1}$.

channel $\mathbf{h_2} = (1/\sqrt{44})[1 \quad 2 \quad 3 \quad 4 \quad 3 \quad 2 \quad 1]^T$ investigated in [6], [8] is used for the performance evaluation. Furthermore, an extremely severe ISI channel $\mathbf{h_3} = (1/\sqrt{85})[1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 4 \quad 3 \quad 2 \quad 1]^T$ is also introduced for evaluating the performance of iterative equalizers.

The legend "SISE 1" and "SISE 2" indicate the iterative SISE algorithms which are described in Section IV. Specifically, TI-BiDFE algorithm of [8] is adopted as the main equalizer and TI-LE of [3] is used for the branch equalizer. Each DFE in BiDFE consists of 21 feedforward taps and 6 feedback taps for $\mathbf{h_2}$ while 21 feedforward taps and 8 feedback taps for $\mathbf{h_3}$. The LE uses 27 taps for $\mathbf{h_2}$ and 29 taps for $\mathbf{h_3}$. The "BiDFE" method is the iterative BiDFE algorithm which does not adopt the branch equalizer LE, simulated for performance comparison purposes. "MAP" is the optimal equalizer implemented via the BCJR algorithm. Finally, the decoder is implemented using the BCJR algorithm and 20 turbo outer iterations are applied to achieve the full potential of each turbo equalization systems.

Fig. 5 shows the performance of several turbo equalizers on the ISI channel $\mathbf{h_2}$. As seen in the figure, the SISE algorithms consistently show the better performance than the single BiDFE with any filter types does. When the number of filter taps increases to 82 (35 feedforward and 6 feedback filter taps for each DFE) versus a total of 54, the TI-BiDFE method does not provide any performance gain, indicating that using 54 taps for this channel already realizes TI-BiDFE's full potential. Moreover, when the same set of simulation is applied to the extremely severe ISI channel, $\mathbf{h_3}$, where the single BiDFE fails to operate adequately because the erroneously generated *a priori* LLRs from the decoder during the iterations cause more errors in the subsequent turbo iterations, the proposed SISE algorithms still show good performance in Fig. 6.

The performance of the turbo equalizers are analyzed by using the extrinsic information transfer (EXIT) chart of [15]. In order to avoid excessive cluttering, the trajectories of only "SISE 1", "SISE 2", and its single main equalizer are plotted

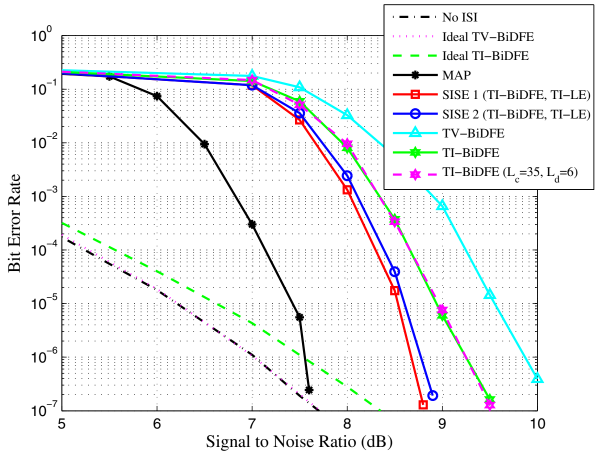

Fig. 5: BER Curves on the Channel $\mathbf{h_2}$ after 20 outer iterations.

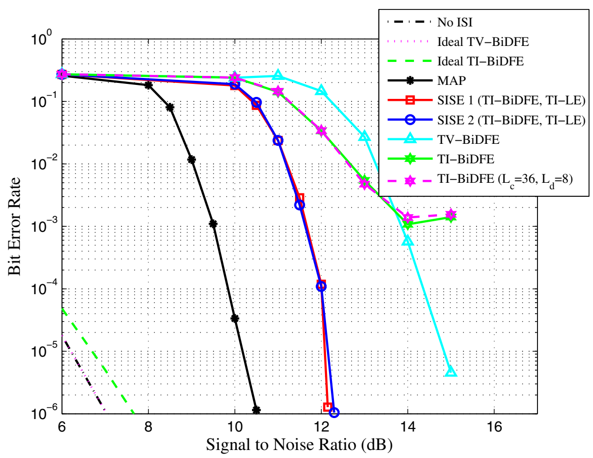

Fig. 6: BER Curves on the Channel $\mathbf{h_3}$ after 20 outer iterations.

in Fig. 7. It describes the EXIT charts on $\mathbf{h_3}$ at a 13 dB SNR. As the figure shows, both SISE algorithms widen the EXIT chart tunnel with aid of the branch equalizers while the trajectory of the single main equalizer itself tends to be stuck at the bottleneck region. Accordingly, SISE algorithms reach the maximum mutual information value at a considerably smaller number of steps than the single BiDFE scheme.

VI. Conclusion

In this paper, we proposed self-iterating soft equalizers which can be employed in turbo equalization systems for a further performance enhancement. The proposed algorithms are designed to utilize the extrinsic information of other "serially" concatenated suboptimal equalizers by reducing correlation on the information generated by other equalizers. The proposed algorithms clearly show their advantages especially when the single suboptimal equalizers are individually weak, such as when they operate on severe ISI channels. Moreover, these algorithms also provide good BER performance in uncoded systems.

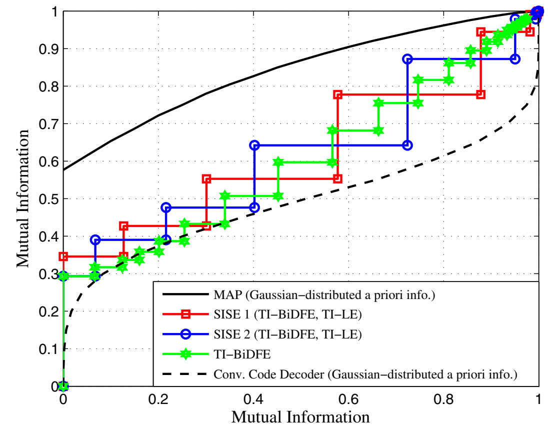

Fig. 7: EXIT Chart on the Channel $\mathbf{h_3}$ at a 13 dB.

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