MMSE-Based Filter Design for Multi-User Peer-to-Peer MIMO Amplify-and-Forward Relay Systems

Joonwoo Shin*, Jaekyun Moon† and Jaeyoung Ahn*

†EE Department, Korea Advanced Institute of Science and Technology (KAIST), Daejon, Korea
*Electronics and Telecommunication Research Institute (ETRI), Daejon, Korea
Email: {joonoos, jyahn}@etri.re.kr, jmoon@kaist.edu

***Abstract*—[1]This paper is concerned with linear relay and destination filter design methods for the multi-user peer-to-peer amplify-and-forward relaying systems. Specifically, the relay and destination filter sets are developed which minimize the sum mean-squared-error (MSE). We first present a joint optimum relay and destination filter calculation method with an iterative algorithm. Motivated by the need to reduce computational complexity of the iterative scheme, we then formulate a simplified sum MSE minimization problem using the relay filter decomposability, which lead to two sub-optimum non-iterative design methods. One is based on zero-forcing channel-inversion and the other on minimum-mean-squared-error channel-inversion. Finally, we propose modified destination filter design methods which require only local channel state information between relay and a specific destination node. The simulation results verify that, compared with the optimum iterative method, the proposed non-iterative schemes suffer a marginal loss in performance while enjoying significantly improved implementation efficiencies.**

I. Introduction

To improve the capacity and/or expand the coverage of wireless networks, multiple-input multiple-output (MIMO) wireless communication systems assisted by an amplify-and-forward (AF) relay have been extensively studied [1]–[13]. While such MIMO AF-relay-assisted schemes were originally developed with a single source, relay and destination in mind [1]–[3], the growing interest in the broad potential applications of wireless ad hoc networks has motivated increasing research efforts in developing efficient cooperative communication schemes for multi-user peer-to-peer wireless relay networks [4]–[13].

In such multi-user peer-to-peer wireless relay networks, transmit-power-minimizing relay design methods with quality-of-service have been studied in multiple relay models [4]–[6]. Taking into account synchronization and channel information exchanges between relays, single MIMO relay schemes which are modelled by a multiple-antenna relay or a collection of distributed antennas that are wired together become advantageous over schemes that consist of multiple independent relays [7]. For such a multi-user single MIMO relay system, transmit-power-minimizing relay design methods with an signal-to-interference-and-noise ratio (SINR) constraint have been investigated [8]–[10], and sum-rate-maximizing relay design methods have been proposed with zero-forcing (ZF) beamforming [7] and block-wise channel decomposition [11]. As an alternative design criterion, the mean-squared-error (MSE) in symbol estimation can be used to guarantee communication link reliability. For single-antenna multi-user (where each source-destination pair is equipped with one transmit antenna and one receive antenna) wireless relay systems, sum-MSE-minimizing relay design methods have been proposed [12], [13]. Specifically, the authors of [12] have proposed a *joint* relay and destination filter design method with an iterative algorithm, and the authors of [13] have studied *separate* relay, source and destination filter design methods.

In this paper, we consider multiple-antenna[2] multi-user systems with a single MIMO AF relay. We provide sum-MSE-minimizing linear relay and destination filter design strategies. We first develop a joint optimum relay and destination filter design method. Due to the non-convexity of the joint optimization problem, we propose an iterative method which is guaranteed to converge to a local optimum point. Compared with [12], where the relay precoder has been calculated with an interior-point method which does not provide an analytical solution, we provide an analytical solution with Karush-Kuhn-Tucker (KKT) conditions. Furthermore, in an effort to reduce the computational complexity of the iterative method, we also pursue non-iterative methods. For this, we formulate a simplified sum-MSE-minimizing problem exploiting the minimum-mean-squared-error (MMSE) relay filter decomposability. Because the cross-link interference from relay to destination nodes makes it difficult to find analytical solutions, we adopt a general channel inversion method used in the multiuser broadcast channel [14]. Then we find a zero-forcing channel-inversion (ZF-CI) based a non-iterative sub-optimum precoder design method. To improve the low-to-mid signal-to-noise ratio (SNR) region performance, we also suggest a minimum-mean-squared-error channel-inversion (MMSE-CI) based non-iterative method. Besides, to avoid the global channel state information (CSI)[3] requirement for MMSE destination filters, we provide modified destination filter design methods relying only on the local CSI, the channel information from relay to

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[2]Compared with single-antenna multi-user systems in [12], [13], multiple antennas support spatial multiplexing between each source-destination pair

[3]Cascaded channel information from all source nodes to a specific destination node via relay

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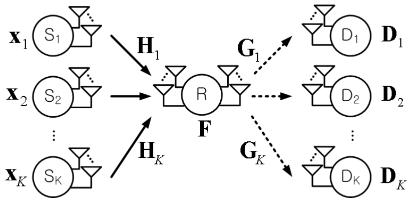

Fig. 1. Multi-user peer-to-peer MIMO AF relay system

a specific destination node. Through numerical simulation, we verify the bit error rate (BER) and sum MSE performance of the proposed schemes. Also, we confirm that the proposed non-iterative method, specifically the MMSE-CI-based sub-optimal scheme, suffers little performance loss at all SNR values in comparison with the optimal iterative design method. In addition, we confirm that the local-CSI-based modification to the destination filters does not induce any noticeable performance loss.

The following notations are used. We employ upper case boldface letters for matrices and lower case boldface letters for vectors. For any general matrix $\mathbf{X}$, $\mathbf{X}^T$, $\mathbf{X}^*$, $\mathbf{X}^H$, $\text{Tr}(\mathbf{X})$, $\text{SVD}(\mathbf{X})$, $\text{Blkdiag}\big[\{\mathbf{X}_i\}\big]$, $\big[\{\mathbf{X}_{ij}\}\big]$ denote the transpose, the conjugate, the Hermitian transpose, the trace, the singular value decomposition of $\mathbf{X}$, the block diagonal matrix with $\{\mathbf{X}_i\}$ as its diagonal matrix block, and the block matrix with $\mathbf{X}_{ij}$ as the (i,j)-th element matrix block, respectively. The symbol $||\cdot||_2$ indicates the 2-norm of a vector. An identity matrix of size n is denoted by $\mathbf{I}_n$.

II. System model

We consider multi-user peer-to-peer MIMO AF relay systems where there are K source-destination terminal pairs and a single relay node as shown in Fig.1. Here, the source, destination and relay nodes are equipped with N_s, N_d, and N_r antennas, respectively. In this system, the k-th source node S_k $(k = 1 \sim K)$ wants to send information to the k-th destination node D_k, and the relay node R helps the communication between each pair. Here we assume that there is no direct link between the source and destination nodes due to a relatively large path loss compared to the links via relay. We consider a spatial multiplexing system in which each source node transmits $M = \min(N_s, N_d)$ data streams simultaneously (meaning that the number of total transmit data streams is KM). To process KM data streams at relay, we assume $N_r \geq KM$. The MIMO channels from S_k to R and R to D_k are modelled by $\mathbf{H}_k \in \mathcal{C}^{N_r \times N_s}$ and $\mathbf{G}_k \in \mathcal{C}^{N_d \times N_r}$, respectively. The transmit signal vector $\mathbf{x}_k \in \mathcal{C}^{N_s \times 1}$ satisfies $\mathbb{E}\{\mathbf{x}_k\mathbf{x}_k^H\} = \sigma_{x,k}^2\mathbf{I}_{N_s}$ and $\sigma_{x,k}^2 = P_k/N_s$. For compact presentation, we assume that all source nodes have equal transmit power, i.e., $P_k = P_s \ \forall\ k$. Note that it is straightforward to consider extension to the unequal transmit power case. In the first time slot, K source nodes $\{\mathsf{S}_k\}$ transmit their signals $\{\mathbf{x}_k\}$ to the relay node simultaneously. The relay node receives a signal vector $\mathbf{r} \in \mathcal{C}^{N_r \times 1}$ as given by $\mathbf{r} = \sum_{i=1}^K \mathbf{H}_i\mathbf{x}_i + \mathbf{n}_r$ where $\mathbf{n}_r$ denotes the complex Gaussian noise vector with zero mean and $\mathbb{E}\{\mathbf{n}_r\mathbf{n}_r^H\} = \sigma_r^2\mathbf{I}_{N_r}$. In the second time slot, the relay node multiplies the received signal $\mathbf{r}$ with a relay filter $\mathbf{F} \in \mathcal{C}^{N_r \times N_r}$. Then, the relay node transmits a signal vector given by $\mathbf{x}_r = \mathbf{F}\mathbf{r} = \sum_{i=1}^K \mathbf{F}\mathbf{H}_i\mathbf{x}_i + \mathbf{F}\mathbf{n}_r$ where the transmit power constraint at the relay is expressed as

$$\text{Tr}\Big(\mathbf{F}(\sum_{i=1}^{K} \sigma_x^2\mathbf{H}_i\mathbf{H}_i^H + \sigma_r^2\mathbf{I}_{N_r})\mathbf{F}^H\Big) = P_R, \tag{1}$$

with P_R denoting the maximum relay transmit power. Finally, the received signal vector at D_k is expressed as

$$\mathbf{y}_k = \mathbf{G}_k\mathbf{F}\mathbf{H}_k\mathbf{x}_k + \sum_{i\neq k}\mathbf{G}_k\mathbf{F}\mathbf{H}_i\mathbf{x}_i + \mathbf{G}_k\mathbf{F}\mathbf{n}_r + \mathbf{n}_k \tag{2}$$

where $\mathbf{n}_k$ is the complex Gaussian noise vector with zero mean and $\mathbb{E}\{\mathbf{n}_k\mathbf{n}_k^H\} = \sigma_d^2\mathbf{I}_{N_d}$. Applying the receive filter $\mathbf{D}_k \in \mathcal{C}^{N_s \times N_d}$, the estimated signal $\hat{\mathbf{x}}_k$ at D_k is expressed as

$$\hat{\mathbf{x}}_k = \mathbf{D}_k\mathbf{G}_k\mathbf{F}\mathbf{H}_k\mathbf{x}_k + \mathbf{D}_k\sum_{i\neq k}\mathbf{G}_k\mathbf{F}\mathbf{H}_i\mathbf{x}_i + \mathbf{D}_k\mathbf{G}_k\mathbf{F}\mathbf{n}_r + \mathbf{D}_k\mathbf{n}_k. \tag{3}$$

III. Iterative filter design

In this section, we investigate the optimum relay and destination filters in terms of sum MSE minimization, first assuming that both $\{\mathbf{H}_k\}$ and $\{\mathbf{G}_k\}$ are known perfectly at relay as well as at destination nodes:

$$\Big\{\{\mathbf{D}_k^\star\},\mathbf{F}^\star\Big\} = \arg\min \sum_{i=1}^{K} \text{Tr}(\mathbf{E}_i) \tag{4}$$

$$\text{subject to } \text{Tr}\Big(\mathbf{F}(\sum_{i=1}^{K} \sigma_x^2\mathbf{H}_i\mathbf{H}_i^H + \sigma_r^2\mathbf{I}_{N_r})\mathbf{F}^H\Big) = P_R$$

where $\mathbf{E}_i$ is the error covariance matrix as given by $\mathbf{E}_i = \mathbb{E}\{(\mathbf{x}_i - \hat{\mathbf{x}}_i)(\mathbf{x}_i - \hat{\mathbf{x}}_i)^H\}$. It is easy to verify that this problem is strictly convex with respect to each of $\mathbf{F}$ and $\mathbf{D}_k \ \forall\ k$, although it is generally non-convex in the joint optimization perspective. Therefore we propose an iterative method that successively optimizes one of $\mathbf{D}_k \ \forall\ k$ and $\mathbf{F}$ while fixing the others.

At first, for given $\mathbf{F}$, the optimum receive filter at D_k (5) is simply derived from (4). Likewise, for given $\{\mathbf{D}_k\}$, the optimum relay filter can be derived. To convert (4) into an unconstrained minimization problem, let us denote $\mathbf{F} = \beta\overline{\mathbf{F}}$, where β is defined as

$$\beta = \sqrt{\frac{P_R}{\text{Tr}\Big(\overline{\mathbf{F}}(\sum_{i=1}^K \sigma_x^2\mathbf{H}_i\mathbf{H}_i^H + \sigma_r^2\mathbf{I}_{N_r})\overline{\mathbf{F}}^H\Big)}}. \tag{6}$$

Then we substitute $\beta\overline{\mathbf{F}}$ for $\mathbf{F}$ in the objective function of (4). Now the problem of computing the optimum filter matrix can be written as

$$\overline{\mathbf{F}}^\star = \arg\min \sum_{i=1}^{K} \mathbb{E}||\mathbf{x}_i - \beta^{-1}\hat{\mathbf{x}}_i||_2^2 \tag{7}$$

$$\text{subject to } \beta^2\text{Tr}\Big(\overline{\mathbf{F}}(\sum_{i=1}^{K} \sigma_x^2\mathbf{H}_i\mathbf{H}_i^H + \sigma_r^2\mathbf{I}_{N_r})\overline{\mathbf{F}}^H\Big) = P_R$$

$$\mathbf{D}_k^\star = \mathbf{H}_k^H\mathbf{F}^H\mathbf{G}_k^H(\sum_{i=1}^K \mathbf{G}_k\mathbf{F}\mathbf{H}_i\mathbf{H}_i^H\mathbf{F}^H\mathbf{G}_k^H + \frac{\sigma_r^2}{\sigma_x^2}\mathbf{G}_k\mathbf{F}\mathbf{F}^H\mathbf{G}_k^H + \frac{\sigma_d^2}{\sigma_x^2}\mathbf{I}_{N_d})^{-1} \tag{5}$$

$$\overline{\mathbf{F}}^\star = \sum_{k=1}^K \Big\{ \Big(\sum_{i=1}^K \mathbf{G}_i^H\mathbf{D}_i^H\mathbf{D}_i\mathbf{G}_i + P_R^{-1}\big(\sum_{l=1}^K \sigma_d^2 \mathrm{Tr}(\mathbf{D}_l\mathbf{D}_l^H)\big)\mathbf{I}_{N_r}\Big)^{-1}\mathbf{G}_k^H\mathbf{D}_k^H\mathbf{H}_k^H\Big(\sum_{j=1}^K \mathbf{H}_j\mathbf{H}_j^H + \frac{\sigma_r^2}{\sigma_x^2}\mathbf{I}_{N_r}\Big)^{-1}\Big\} \tag{8}$$

From the KKT conditions [15], the solution of (7) is represented as (8), and $\mathbf{F}^\star = \beta\overline{\mathbf{F}}^\star$ can be obtained with (6).

With the above $\{\mathbf{D}_k^\star\}$ and $\mathbf{F}^\star$, we propose an iterative algorithm for solving (4). Note that even though Algorithm 1 cannot guarantee the global optimal solution due to the non-convexity of the minimization problem (4), a local optimum point can be found. Also Algorithm 1 is provably convergent (proof will not be given due to the space limitation).

Algorithm 1 Obtaining optimal relay and destination filters

Set $l = 0$. Initialize $\overline{\mathbf{F}}$ with an arbitrary $N_r \times N_r$ matrix. Calculate $\{\mathbf{D}_k\}$ with the initial matrices.
repeat
  $l := l + 1$
  Step 1: Update β and $\overline{\mathbf{F}}$ with (6), (8).
  Step 2: Update $\{\mathbf{D}_k\}$ with (5).
  Step 3: Calculate $\mathrm{SMSE}_{(l)}$ $(= \sum_{i=1}^K \mathrm{Tr}(\mathbf{E}_i))$ with the updated matrices.
until $|\mathrm{SMSE}_{(l)} - \mathrm{SMSE}_{(l-1)}| < \epsilon$, where ϵ is the arbitrarily small value.

IV. Non-iterative filter design

Compared to the above iterative relay and destination filter matrix identification method, non-iterative methods are computationally efficient. In this section, we pursue non-iterative filter design methods.

A. Simplified optimization problem

Lemma 1: For given $\{\mathbf{D}_k^\star\}$, the optimum relay filter $\mathbf{F}^\star$ for minimizing the sum MSE has the following form: $\mathbf{F}^\star = \sum_{k=1}^K \mathbf{B}_k\mathbf{W}_k$, where $\mathbf{B}_k$ and $\mathbf{W}_k$ are computed as follows:

$$\mathbf{B}_k = \Big(\sum_{i=1}^K \mathbf{G}_i^H\mathbf{D}_i^H\mathbf{D}_i\mathbf{G}_i + \lambda\mathbf{I}_{N_r}\Big)^{-1}\mathbf{G}_k^H\mathbf{D}_k^H \tag{9a}$$

$$\mathbf{W}_k = \mathbf{H}_k^H\big(\sum_{j=1}^K \mathbf{H}_j\mathbf{H}_j^H + \frac{\sigma_r^2}{\sigma_x^2}\mathbf{I}_{N_r}\big)^{-1} \tag{9b}$$

where λ is the Lagrange multiplier to satisfy the relay transmit power constraint.

Proof: Proof will be given elsewhere due to space limitations. ■

Note that Lemma 1 is the generalization of [16] in the single-user MIMO AF relaying systems to the multi-user peer-to-peer MIMO AF relaying system.

With MMSE receive filters $\{\mathbf{D}_k\}$ and Lemma 1, sum MSE in (4) can be rewritten as (10) below (see Appendix A for derivations), where $\boldsymbol{\Upsilon}_i \triangleq \mathbf{B}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{B}$ with $\mathbf{B} = [\mathbf{B}_1, \cdots, \mathbf{B}_K]$, $\mathbf{W} = [\mathbf{W}_1^T, \cdots, \mathbf{W}_K^T]^T$ and $\mathbf{R} = [\{\mathbf{R}_{ij}\}] = \sigma_x^2\mathbf{W}\big(\sum_{i=1}^K \mathbf{H}_i\mathbf{H}_i^H + \frac{\sigma_r^2}{\sigma_x^2}\mathbf{I}_{N_r}\big)\mathbf{W}^H$, $\mathbf{R}_{ij} = \sigma_x^2\mathbf{W}_i\big(\sum_{k=1}^K \mathbf{H}_k\mathbf{H}_k^H + \frac{\sigma_r^2}{\sigma_x^2}\mathbf{I}_{N_r}\big)\mathbf{W}_j^H$, $i, j \in \{1 \sim K\}$. Note that because the first summation term on the right hand side of (10) depends only on the source to relay channels $\{\mathbf{H}_i\}$, it needs not be considered during optimization. Now the simplified optimization problem of (4) is formulated as (note that we presume MMSE destination filters $\{\mathbf{D}_k^\star\}$):

$$\min_{\mathbf{B}} \sum_{i=1}^K \mathrm{Tr}(\mathbf{H}_i^H\mathbf{W}^H\mathbf{R}^{-1}(\mathbf{R}^{-1} + \sigma_d^{-2}\boldsymbol{\Upsilon}_i)^{-1}\mathbf{R}^{-1}\mathbf{W}\mathbf{H}_i)$$
$$\text{subject to } \mathrm{Tr}(\mathbf{B}\mathbf{R}\mathbf{B}^H) = P_R. \tag{11}$$

The power constraint of (11) has been arranged as $\mathrm{Tr}(\mathbf{F}(\sum_{i=1}^K \sigma_x^2\mathbf{H}_i\mathbf{H}_i^H + \sigma_r^2\mathbf{I}_{N_r})\mathbf{F}^H) = \mathrm{Tr}(\mathbf{B}\mathbf{W}(\sum_{i=1}^K \sigma_x^2\mathbf{H}_i\mathbf{H}_i^H + \sigma_r^2\mathbf{I}_{N_r})\mathbf{W}^H\mathbf{B}^H) = \mathrm{Tr}(\mathbf{B}\mathbf{R}\mathbf{B}^H)$. The analytical optimal solution to this problem is difficult to derive due to the cross-link interference from the relay node to the destination nodes; we resort to efficient sub-optimal approaches.

B. ZF-CI-based suboptimum strategy

To eliminate the cross-link interference from the relay node to the destination nodes, we make the blockwise forward channel splitting, i.e., $\mathbf{G}_i\mathbf{B}_j = \mathbf{0}$ $(i \neq j \in \{1 \sim K\})$, assuming $\mathbf{B}_i = \mathbf{Q}_i^{(\mathrm{ZF})}\mathbf{M}_i^{(\mathrm{ZF})}$ $\forall i$, where $\mathbf{Q}_i^{(\mathrm{ZF})} \in \mathcal{C}^{N_r \times N_s}$ forms an orthonormal basis for the null space of $[\mathbf{G}_1^T, \cdots, \mathbf{G}_{i-1}^T, \mathbf{G}_{i+1}^T, \cdots, \mathbf{G}_K^T]^T$, and $\mathbf{M}_i^{(\mathrm{ZF})} \in \mathcal{C}^{N_s \times N_s}$ is the post-processor to be optimized. As in the generalized ZF-CI method in [14], $\mathbf{Q}_i^{(\mathrm{ZF})}$ can be obtained from QR-decomposition, $\mathbf{G}_i^{(\mathrm{ZF})} = \mathbf{Q}_i^{(\mathrm{ZF})}\mathbf{T}_i^{(\mathrm{ZF})} \in \mathcal{C}^{N_r \times N_s}$, where $\mathbf{T}_i^{(\mathrm{ZF})}$ is an $N_s \times N_s$ upper-triangular matrix, $\mathbf{G}^H(\mathbf{G}\mathbf{G}^H)^{-1} = [\mathbf{G}_1^{(\mathrm{ZF})}, \cdots, \mathbf{G}_K^{(\mathrm{ZF})}]$, and $\mathbf{G} = [\mathbf{G}_1^T, \cdots, \mathbf{G}_K^T]^T$. For ease of derivation, we approximate $\mathbf{R} \approx \mathbf{I}_{N_r}$ [4]. Then the modified objective function is written as (12) below (see Appendix B for derivations), where $\mathbf{W}\mathbf{H}_k\mathbf{H}_k^H\mathbf{W}^H = [\{\mathbf{C}_{ij}^{(k)}\}]$, $\mathbf{C}_{ij}^{(k)} \in \mathcal{C}^{N_s \times N_s}$, and $i, j \in \{1 \sim K\}$. Because the first summation term in (12) is irrelevant to $\{\mathbf{M}_i^{(\mathrm{ZF})}\}$, the optimization problem

[4]Our simulation results indicate that this approximation is reasonable.

$$\sum_{i=1}^{K}\text{Tr}(\mathbf{E}_i)=\sum_{i=1}^{K}\sigma_x^2\text{Tr}(\mathbf{I}_{N_s}-\sigma_x^2\mathbf{H}_i^H\mathbf{W}^H\mathbf{R}^{-1}\mathbf{W}\mathbf{H}_i)+\sum_{i=1}^{K}\sigma_x^4\text{Tr}(\mathbf{H}_i^H\mathbf{W}^H\mathbf{R}^{-1}(\mathbf{R}^{-1}+\sigma_d^{-2}\boldsymbol{\Upsilon}_i)^{-1}\mathbf{R}^{-1}\mathbf{W}\mathbf{H}_i) \tag{10}$$

$$\min_{\{\mathbf{M}_i^{(\text{ZF})}\}}\sum_{i=1}^{K}\Big\{\sum_{j\neq i}^{K}\text{Tr}\{\mathbf{C}_{jj}^{(i)}\}+\text{Tr}\{\mathbf{C}_{ii}^{(i)}(\mathbf{I}_{N_r}+\sigma_d^{-2}{\mathbf{M}_i^{(\text{ZF})}}^H{\mathbf{Q}_i^{(\text{ZF})}}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{Q}_i^{(\text{ZF})}\mathbf{M}_i^{(\text{ZF})})^{-1}\}\Big\} \tag{12}$$

(12) is simplified as:

$$\min_{\{\mathbf{M}_i^{(\text{ZF})}\}}\sum_{i=1}^{K}\text{Tr}\{\mathbf{C}_{ii}^{(i)}(\mathbf{I}_{N_r}+\sigma_d^{-2}{\mathbf{M}_i^{(\text{ZF})}}^H\boldsymbol{\Xi}_i^{(\text{ZF})}\mathbf{M}_i^{(\text{ZF})})^{-1}\}$$
$$\text{subject to }\sum_{i=1}^{K}\text{Tr}(\mathbf{R}_{ii}{\mathbf{M}_i^{(\text{ZF})}}^H\mathbf{M}_i^{(\text{ZF})})=P_R, \tag{13}$$

where $\boldsymbol{\Xi}_i^{(\text{ZF})}\triangleq{\mathbf{Q}_i^{(\text{ZF})}}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{Q}_i^{(\text{ZF})}$.

Now in order to decouple the block channel $\boldsymbol{\Xi}_i^{(\text{ZF})}$ into N_s parallel sub-channels, the singular value decomposition of $\boldsymbol{\Xi}_i^{(\text{ZF})}$ is computed as

$$\text{SVD}(\boldsymbol{\Xi}_i^{(\text{ZF})})=\mathbf{U}_i^{(\text{ZF})}\boldsymbol{\Lambda}_i^{(\text{ZF})}{\mathbf{U}_i^{(\text{ZF})}}^H. \tag{14}$$

Then, employing $\mathbf{M}_i^{(\text{ZF})}=\mathbf{U}_i^{(\text{ZF})}{\boldsymbol{\Gamma}_i^{(\text{ZF})}}^{1/2}$ without loss of generality, the power loading matrix $\boldsymbol{\Gamma}_i^{(\text{ZF})}=\text{diag}(\gamma_{i1}^{(\text{ZF})},\cdots,\gamma_{iN_s}^{(\text{ZF})})$ is the only optimization parameter. Finally, the optimization problem (13) is rewritten as:

$$\min_{\{\boldsymbol{\Gamma}_i^{(\text{ZF})}\}}\sum_{i=1}^{K}\text{Tr}\{\mathbf{C}_{ii}^{(i)}(\mathbf{I}_{N_r}+\sigma_d^{-2}\boldsymbol{\Lambda}_i^{(\text{ZF})}\boldsymbol{\Gamma}_i^{(\text{ZF})})^{-1}\}$$
$$\text{subject to }\sum_{i=1}^{K}\text{Tr}(\mathbf{R}_{ii}\boldsymbol{\Gamma}_i^{(\text{ZF})})=P_R. \tag{15}$$

This problem can be solved with KKT conditions. Denoting c_{ij} and ρ_{ij} as the j-th diagonal elements of $\mathbf{C}_{ii}^{(i)}$ and $\mathbf{R}_{ii}$, respectively, we obtain the optimal solution $\gamma_{ij}^{(\text{ZF})}=\max\Big(\sqrt{\frac{c_{ij}}{\mu^{(\text{ZF})}\rho_{ij}\sigma_d^{-2}\lambda_{ij}^{(\text{ZF})}}}-\frac{1}{\sigma_d^{-2}\lambda_{ij}^{(\text{ZF})}},0\Big)$, where $\mu^{(\text{ZF})}$ is the water level chosen to satisfy the power constraint $\sum_{i=1}^{K}\sum_{j=1}^{N_s}\rho_{ij}\gamma_{ij}^{(\text{ZF})}=P_R$ and $\lambda_{ij}^{(\text{ZF})}$ is the j-th diagonal element of $\boldsymbol{\Lambda}_i^{(\text{ZF})}$. Then from Lemma 1, the ZF-CI-based suboptimum relay filter $\mathbf{F}^{(\text{ZF})}=\sum_{k=1}^{K}\mathbf{B}_k^{(\text{ZF})}\mathbf{W}_k$ is given by $\mathbf{B}_k^{(\text{ZF})}=\mathbf{Q}_k^{(\text{ZF})}\mathbf{U}_k^{(\text{ZF})}{\boldsymbol{\Gamma}_k^{(\text{ZF})}}^{1/2}$.

C. MMSE-CI-based suboptimum strategy

Due to the complete suppression of cross-link interference at the expense of noise enhancement, performance degradation of the ZF-CI-based strategy at the low-to-medium SNR region is inevitable. To improve the low-to-mid SNR region performance, in this subsection, we investigate the MMSE-CI strategy. Similar to the ZF-CI-based scheme, we assume $\mathbf{B}_i=\mathbf{Q}_i^{(\text{MMSE})}\mathbf{M}_i^{(\text{MMSE})}\ \forall i$, where $\mathbf{Q}_i^{(\text{MMSE})}$ is obtained from QR-decomposition, $\mathbf{G}_i^{(\text{MMSE})}=\mathbf{Q}_i^{(\text{MMSE})}\mathbf{T}_i^{(\text{MMSE})}$. To reduce the effect of noise, we set $\mathbf{G}^H(\mathbf{G}\mathbf{G}^H+\alpha\mathbf{I})^{-1}=[\mathbf{G}_1^{(\text{MMSE})},\cdots,\mathbf{G}_K^{(\text{MMSE})}]$ with α chosen as $K\sigma_d^2/P_R$ [17]. Unlike the ZF method, $\mathbf{B}_i\ (=\mathbf{Q}_i^{(\text{MMSE})}\mathbf{M}_i^{(\text{MMSE})})$ constructed by a linear combination of $\mathbf{Q}_i^{(\text{MMSE})}$ generates residual interference. These residual interference makes it difficult to find an analytical solution. For deriving an MMSE-CI-based solution, we assume a high SNR regime, i.e., $\mathbf{G}_i\mathbf{Q}_j^{(\text{MMSE})}\approx\mathbf{0}\ (i\neq j)$ [5]. Then the MMSE solution is given by

$$\min\sum_{i=1}^{K}\text{Tr}\{\mathbf{C}_{ii}^{(i)}(\mathbf{I}_{N_r}+\sigma_d^{-2}{\mathbf{M}_i^{(\text{MMSE})}}^H\boldsymbol{\Xi}_i^{(\text{MMSE})}\mathbf{M}_i^{(\text{MMSE})})^{-1}\}$$
$$\text{subject to }\sum_{i=1}^{K}\text{Tr}(\mathbf{R}_{ii}{\mathbf{M}_i^{(\text{MMSE})}}^H\mathbf{M}_i^{(\text{MMSE})})=P_R, \tag{16}$$

where $\boldsymbol{\Xi}_i^{(\text{MMSE})}\triangleq{\mathbf{Q}_i^{(\text{MMSE})}}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{Q}_i^{(\text{MMSE})}$. This problem is identical to (13) if $\mathbf{M}_i^{(\text{MMSE})}$ and $\mathbf{Q}_i^{(\text{MMSE})}$ are replaced by $\mathbf{M}_i^{(\text{ZF})}$ and $\mathbf{Q}_i^{(\text{ZF})}$, respectively. The optimum solutions $\mathbf{M}_i^{(\text{MMSE})}=\mathbf{U}_i^{(\text{MMSE})}{\boldsymbol{\Gamma}_i^{(\text{MMSE})}}^{1/2}$ are derived in the same way as in the ZF-CI method, using $\text{SVD}(\boldsymbol{\Xi}_i^{(\text{MMSE})})=\mathbf{U}_i^{(\text{MMSE})}\boldsymbol{\Lambda}_i^{(\text{MMSE})}{\mathbf{U}_i^{(\text{MMSE})}}^H$, $\boldsymbol{\Gamma}_i^{(\text{MMSE})}=\text{diag}(\gamma_{i1}^{(\text{MMSE})},\cdots,\gamma_{iN_s}^{(\text{MMSE})})$, and $\gamma_{ij}^{(\text{MMSE})}=\max\Big(\sqrt{\frac{c_{ij}}{\mu^{(\text{MMSE})}\rho_{ij}\sigma_d^{-2}\lambda_{ij}^{(\text{MMSE})}}}-\frac{1}{\sigma_d^{-2}\lambda_{ij}^{(\text{MMSE})}},0\Big)$, where $\mu^{(\text{MMSE})}$ is chosen to satisfy the power constraint $\sum_{i=1}^{K}\sum_{j=1}^{N_s}\rho_{ij}\gamma_{ij}^{(\text{MMSE})}=P_R$ and $\lambda_{ij}^{(\text{MMSE})}$ is the j-th diagonal element of $\boldsymbol{\Lambda}_i^{(\text{MMSE})}$. In the end, the MMSE-CI-based suboptimum relay filter $\mathbf{F}^{(\text{MMSE})}=\sum_{k=1}^{K}\mathbf{B}_k^{(\text{MMSE})}\mathbf{W}_k$ is found with $\mathbf{B}_k^{(\text{MMSE})}=\mathbf{Q}_k^{(\text{MMSE})}\mathbf{U}_k^{(\text{MMSE})}{\boldsymbol{\Gamma}_k^{(\text{MMSE})}}^{1/2}$.

D. Modified destination receive filters using only local channel state information

After finding the suboptimum relay filters, the corresponding destination filters can be found with (5) by replacing $\mathbf{F}$ with the corresponding suboptimum relay filters. We note, however, that to find the destination filters with (5) requires the global channel state information (CSI) at the destination node, corresponding to all cascaded channels from all source nodes to the relay and from relay to corresponding destination nodes. This global CSI requirement is undesirable due to an increased

[5]According to the simulation results, the MMSE-CI method with a high SNR assumption indeed shows considerable performance improvements at the low-to-mid SNR region.

system overhead such as pilot signals. We now provide a modified destination filter design method which requires only the *local* CSI associated with the relay-to-destination paths. For the MMSE receiver in (5), we can obtain a simplified form with the following Lemma.

Lemma 2: Given the optimal relay processing matrix $\mathbf{F}^\star = \mathbf{B}\mathbf{W}$ and the $\mathbf{R} \approx \mathbf{I}$ assumption, the MMSE receiver at the destination (5) can be simplified as

$$\mathbf{D}_k^\star = \mathbf{H}_k^H \mathbf{W}^H (\mathbf{B}^H \mathbf{G}_k^H \mathbf{G}_k \mathbf{B} + \sigma_d^2 \mathbf{I}_{N_r})^{-1} \mathbf{B}^H \mathbf{G}_k^H \quad (17)$$

Proof: Proof will be given elsewhere. ■

Note that Lemma 2 also represents the generalization of the method of [16] to multi-user systems. When the ZF-CI-based suboptimal relay filter is applied, we can obtain a modified receiver using the following Lemma.

Lemma 3: For (17), using the ZF-CI-based relay filter $\mathbf{F}^{(\text{ZF})} = \sum_{i=1}^K \mathbf{B}_i^{(\text{ZF})} \mathbf{W}_i$, we obtain the following modified receiver:

$$\mathbf{D}_k^{(\text{ZF})} = (\mathbf{B}_k^{(\text{ZF})H} \mathbf{G}_k^H \mathbf{G}_k \mathbf{B}_k^{(\text{ZF})} + \sigma_d^2 \mathbf{I}_{N_s})^{-1} \mathbf{B}_k^{(\text{ZF})H} \mathbf{G}_k^H \quad (18)$$

Proof: Proof will be given elsewhere. ■

For the MMSE-CI-based relay $\mathbf{F}^{(\text{MMSE})} = \sum_{i=1}^K \mathbf{B}_i^{(\text{MMSE})} \mathbf{W}_i$, the modified receiver can be written as

$$\begin{aligned}\mathbf{D}_k^{(\text{MMSE})} =& (\mathbf{B}_k^{(\text{MMSE})H} \mathbf{G}_k^H \mathbf{G}_k \mathbf{B}_k^{(\text{MMSE})} + \sigma_d^2 \mathbf{I}_{N_s})^{-1} \\ & \times \mathbf{B}_k^{(\text{MMSE})H} \mathbf{G}_k^H. \end{aligned} \quad (19)$$

Lemma 3 indicates that the destination node does not need to know the channel information $\{\mathbf{H}_i\}$ for source-to-relay paths. In other words, for the proposed suboptimal relay processor design, each destination node needs to know only the local CSI corresponding to its own link. Table I summarizes the required CSI at each nodes including relay node.

	Relay node	Dest. node
Iterative method in section III	$\{\mathbf{H}_i\}, \{\mathbf{G}_i\}$	$\{\mathbf{H}_i\}, \mathbf{G}_k$
Sub-opt. relay with MMSE receiver	$\{\mathbf{H}_i\}, \{\mathbf{G}_i\}$	$\{\mathbf{H}_i\}, \mathbf{G}_k$
Sub-opt. relay with modified receiver	$\{\mathbf{H}_i\}, \{\mathbf{G}_i\}$	$\mathbf{B}_k, \mathbf{G}_k$

TABLE I
COMPARISON OF THE REQUIRED CSI AT THE RELAY NODE AND THE k-TH DESTINATION NODE

V. SIMULATION RESULTS

In this section, we present the numerical results of the proposed schemes in terms of BER and sum MSE. For all simulation runs, we assume spatially uncorrelated flat fading MIMO channels of which elements are generated by independently and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. The average received SNRs at the source-relay link and the relay-destination link are defined as $\text{SNR}_{\text{SR}} = P_s/\sigma_r^2$ and $\text{SNR}_{\text{RD}} = P_R/\sigma_d^2$, respectively. We assume that $K = 3$, $N_s = N_d = 2$, $N_r = 6$ and $\text{SNR}_{\text{SR}} = \text{SNR}_{\text{RD}} = \text{SNR}$. We initialize the iterative method with a power-scaled (to meet the

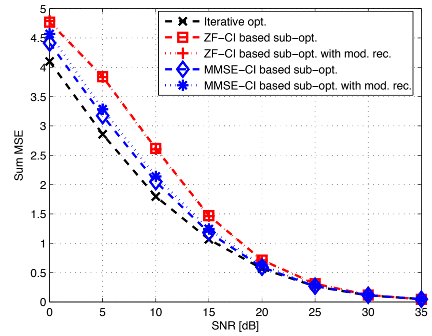

Fig. 2. Sum MSE comparisons between the iterative method and the two non-iterative methods

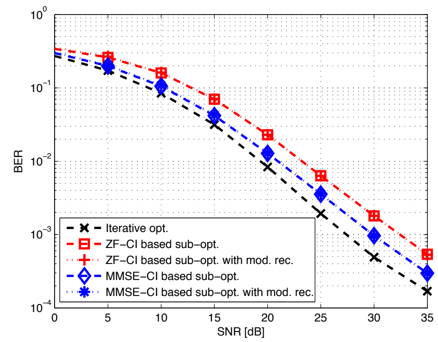

Fig. 3. BER (uncoded) comparisons between the iterative method and the two non-iterative methods (QPSK modulation)

transmit power constraint) random matrix for the relay filter. Fig. 2 shows the sum MSE performance. Note that the MSE curves are obtained after normalizing the MSE by σ_x^2. Clearly the iterative scheme outperforms the two non-iterative schemes because in the iterative scheme the relay has no structural constraint such as ZF-CI or MMSE-CI. Nevertheless, it can be seen that the sum MSE of the ZF-CI-based non-iterative method converges to that of the iterative method as SNR increases. Also the performance of the MMSE-CI-based non-iterative method is close to that of the iterative method in the entire SNR range. The uncoded BER performance comparison of iterative and non-iterative schemes in QPSK constellation are presented in Fig. 3. From this plot, we can also check that the MMSE-CI-based scheme exhibits a marginal performance loss, although the ZF-IC-based scheme shows about 3 dB loss at an uncoded BER of 10^{-2}. It is also worth noting that in the local CSI situation at the destination, the proposed designs leading to the modified receiver nearly achieve the performances of the MMSE receiver that requires global CSI. We also note that the approximation $\mathbf{R} \approx \mathbf{I}_{N_r}$ is strongly validated in the sense of the sum MSE and BER performances. From Figs. 2 and 3, we can confirm that the proposed non-iterative methods, especially the MMSE-CI-based scheme, nearly achieves the performance of optimum iterative schemes.

VI. Conclusion

In this paper, we have studied linear filter design methods for the multi-user peer-to-peer MIMO AF relay system. First, an iterative joint optimum relay and destination filter design method has been developed. Then, two sub-optimum strategies have been proposed, which could be implemented non-iteratively. Also, modified destination filter design methods requiring only the local CSI have been presented. Through numerical simulations, we have demonstrated that the performances of the proposed non-iterative methods approach that of the iterative optimum scheme. The proposed sub-optimum methods have clear advantages in terms of computational complexity and CSI requirements.

Appendix

A. Derivation of the simplified sum MSE

Applying the MMSE receive filters $\{\mathbf{D}_k^\star\}$, the sum MSE (4) can be represented as:

$$\sum_{i=1}^{K}\sigma_x^2\text{Tr}\Big\{\mathbf{I}_{N_s}-\mathbf{H}_i^H\mathbf{F}^H\mathbf{G}_i^H(\mathbf{G}_i\mathbf{F}\mathbf{H}_i\mathbf{H}_i^H\mathbf{F}^H\mathbf{G}_i^H + \mathbf{N}_i)^{-1}\mathbf{G}_i\mathbf{F}\mathbf{H}_i\Big\}, \quad \text{(A.1)}$$

where $\mathbf{N}_i = \sum_{j\neq i}^{K}\mathbf{G}_i\mathbf{F}\mathbf{H}_j\mathbf{H}_j^H\mathbf{F}^H\mathbf{G}_i^H + \frac{\sigma_r^2}{\sigma_x^2}\mathbf{G}_i\mathbf{F}\mathbf{F}^H\mathbf{G}_i^H + \frac{\sigma_d^2}{\sigma_x^2}\mathbf{I}_{N_d}$. Using Lemma 1,

$$\mathbf{G}_i\mathbf{F}\mathbf{H}_i\mathbf{H}_i^H\mathbf{F}^H\mathbf{G}_i^H + \mathbf{N}_i = \frac{1}{\sigma_x^2}\mathbf{G}_i\mathbf{B}\mathbf{R}\mathbf{B}^H\mathbf{G}_i^H + \frac{\sigma_d^2}{\sigma_x^2}\mathbf{I}_{N_d}, \quad \text{(A.2)}$$

where $\mathbf{B}$ and $\mathbf{R}$ are the same as in subsection IV-A. Applying (A.2), the sum MSE (A.1) is rewritten as

$$\sum_{i=1}^{K}\sigma_x^2\text{Tr}\Big\{\mathbf{I}_{N_s}-\mathbf{H}_i^H\mathbf{W}^H\mathbf{B}^H\mathbf{G}_i^H(\mathbf{G}_i\mathbf{B}\mathbf{R}\mathbf{B}^H\mathbf{G}_i^H + \sigma_d^2\mathbf{I}_{N_d})^{-1}\mathbf{G}_i\mathbf{B}\mathbf{W}\mathbf{H}_i\Big\}. \quad \text{(A.3)}$$

With the Woodbury identity [18],

$$(\mathbf{G}_i\mathbf{B}\mathbf{R}\mathbf{B}^H\mathbf{G}_i^H + \sigma_d^2\mathbf{I}_{N_d})^{-1} = \sigma_d^{-2}\mathbf{I}_{N_d} - \sigma_d^{-4}\mathbf{G}_i\mathbf{B}(\mathbf{R}^{-1}+\sigma_d^{-2}\mathbf{B}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{B})^{-1}\mathbf{B}^H\mathbf{G}_i^H, \quad \text{(A.4)}$$

the sum MSE (A.3) reduces to (10).

B. Derivation of the modified cost function (12)

With the approximation $\mathbf{R}\approx\mathbf{I}_{N_r}$, the objective function of (11) is simplified as:

$$\sum_{i=1}^{K}\text{Tr}(\mathbf{H}_i^H\mathbf{W}^H(\mathbf{I}_{N_r}+\sigma_d^{-2}\mathbf{B}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{B})^{-1}\mathbf{W}\mathbf{H}_i) \quad \text{(A.5)}$$

From the fact that $\mathbf{G}_i\mathbf{Q}_j^{(\text{ZF})} = \mathbf{0}$ $(i \neq j)$, $\mathbf{G}_i\mathbf{B} = [\mathbf{0},\cdots,\mathbf{\Omega}_i,\cdots,\mathbf{0}]$ where $\mathbf{\Omega}_i \triangleq \mathbf{G}_i\mathbf{Q}_i^{(\text{ZF})}\mathbf{M}_i^{(\text{ZF})}\mathbf{\Gamma}_i^{(\text{ZF})}$.

$$\begin{aligned}&\big(\mathbf{I}_{N_r}+\sigma_d^{-2}\mathbf{B}^H\mathbf{G}_i^H\mathbf{G}_i\mathbf{B}\big)^{-1}\\ &=\big(\mathbf{I}_{N_r}+\sigma_d^{-2}\text{Blkdiag}\big[\{\mathbf{0},\cdots,\mathbf{\Omega}_i^H\mathbf{\Omega}_i,\cdots,\mathbf{0}\}\big]\big)^{-1}\\ &=\text{Blkdiag}\big[\{\mathbf{I}_{N_s},\cdots,\big(\mathbf{I}_{N_s}+\sigma_d^{-2}\mathbf{\Omega}_i^H\mathbf{\Omega}_i\big)^{-1},\cdots,\mathbf{I}_{N_s}\}\big]\end{aligned} \quad \text{(A.6)}$$

With (A.6), the modified cost function (12) can be obtained from (A.5).

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