Abstract—We present linear transmit-receive filter design strategies for the two-cell multiple-input multiple-output (MIMO) interfering broadcast channel (IBC) where inter-cell interference (ICI) exists in addition to inter-user interference (IUI) within the cell. We first formulate a generalized zero-forcing interference alignment (ZF-IA) method capable of handling multiple data streams for each user. We proceed to further adjust the transmit-receive filters based on the minimum weighted-mean-square-error (WMSE) principle using the weights obtained from the initial generalized ZF-IA design, in an effort to enhance the sum-rate performance. In contrast to the existing weighted-sum-rate-maximizing scheme, our method does not require an iterative calculation of the weights. Because of this, the proposed scheme, while not designed specifically to maximize the sum-rate, is computationally efficient and delivers favorable tradeoffs between performance and information exchange overhead.

I. INTRODUCTION

We consider the MIMO IBC involving two interfering base stations (BSs), which models important wireless network scenarios where the interference from one neighboring cell dominates over all residual interferences from other cells. Various transmitter-receiver (transceiver) strategies have been investigated for the MIMO IBC channel [1]–[4]. An iterative weighted-sum-rate-maximizing transceiver design method for the multi-cell MIMO IBCs has been proposed [1]. An achievable degree of freedom (DoF) has been derived analytically for the two-cell MIMO IBC under no receiver collaboration, and a linear transceiver that attains this DoF has been devised in [2] under the assumption that full global channel state information (CSI) is available at every user node as well as at the BSs. The achievable DoF derived in [2], however, is smaller than the DoF of the MIMO IBC under full user cooperation, under general antenna configurations. Note that the MIMO IBC under full user cooperation can be viewed as a MIMO interference channel for which the DoF is now well-known [5].

To achieve the full DoF of the MIMO IBC, an interference alignment (IA) method is proposed in [4] based on constructing each user’s receive vector along the direction orthogonal to ICI and then cancelling IUI via zero-forcing (ZF) beam-forming at each BS. In this method, the only channel knowledge each receiver needs is that of the interference channel from the neighboring BS to itself, and there is no need for joint receiver processing. Further, the BS in this setting needs to know only the local CSI (i.e., the responses of the channels to its own user nodes) and its users’ ICI-nulling receive vectors.

The authors of [3] have introduced a different variation on IA; in their method, two receivers within a given cell collaborate with each other and first align the ICIIs they are experiencing along a common subspace. The BS then sends signals to its two users along the directions orthogonal to the subspace the ICIIs and IUI lie in. Like the scheme of [4], for beam-forming the BS needs to know the effective channels composed of the CSIs for the intended paths to its own users and the receive vectors of its users. In addition, the BS requires the aligned interference channel response for the paths from itself to the users in the neighboring cell. While the authors of [3] have shown that their approach can achieve the same DoF possible by the method of [4] with a smaller number of antennas, it was not clear whether the former approach could be generalized to the K-user per cell scenario. More recently, however, it has been shown in [6] that the IA approach suggested by [3] can be extended to cover K users per cell. Nevertheless, the methods of [3] and [6] both require collaboration of users within the same cell.

In this letter, we first apply the ZF-IA scheme of [4] to the general case where multiple streams are designated for each user. It is well known that the original MIMO IA method of [7], which has been developed for the MIMO interference channel, is short of maximizing sum-rates at finite SNRs despite its ability to achieve the DoF. Similarly, it is expected that the generalized IA scheme designed for the IBC would not be able to achieve maximum sum-rates at practical SNR values. To this end, we utilize the well-established notion of regularized channel inversion [8] in enhancing the initially designed generalized ZF-IA scheme, in an effort to improve the sum rates at finite SNRs. Through analysis and simulation, we verify that the proposed regularized ZF-IA scheme based on weighted MSE minimization indeed improves upon the generalized ZF-IA method and outperforms the existing weighted sum-rate-maximizing method of [1] if the number of iterations for transceiver filter computation is kept small. The proposed scheme also requires considerably less communication overhead for information exchange between the BSs.

We employ the upper case boldface letters for matrices and the lower case boldface letters for vectors. For any general matrix $X$, $X^*$, $X^H$, $\text{Tr}(X)$, $\text{det}(X)$, and $\text{SVD}(X)$ denote the conjugate, the Hermitian transpose, the trace, the determinant and the singular value decomposition, respectively. The symbol $I_n$ denotes an identity matrix of size $n$. 

J. Shin is with the Electronics and Telecommunications Research Institute (ETRI), Daejeon, Korea (e-mail:joonoos@etri.re.kr). J. Moon is with the EE Department, Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea (e-mail:jmoon@kaist.edu).
II. SYSTEM MODEL

In the two-cell MIMO interfering broadcast channel under consideration, BS $B_m$ is equipped with $M$ antennas, supporting $K$ users $D_{mk}$ in its cell, and each user has $N$ antennas $(m \in (1, 2), k \in (1, \cdots, K))$. $B_m$ sends $L_s$ data streams to each of its $K$ users using $M$ transmit antennas. The signal vector of size $N$ received at $D_{mk}$, $y[m,k]$, is given as

$$y[m,k] = H[m,k]T[m,k]s[m,k] + H[m,k] \sum_{i \neq k}^{K} T[m,i]s[m,i] + H[m,k] \sum_{i=1}^{K} T[m,i]s[m,i] + n[m,k]$$

where $T[m,k] \in C^{M \times L_s}$ is BS $m$’s precoding matrix geared for $D_{mk}$, $s[m,k] \in C^{L_s \times 1}$ stands for the signal vector of length $L_s$ transmitted for $D_{mk}$, $n[m,k] \in C^{N \times M}$ is the Gaussian noise at $D_{mk}$ with $CN(0, \sigma_n^2)$ and $H[m,k] \in C^{N \times M}$ is the channel matrix from $B_m$ to its user $D_{mk}$ whereas $H[m,k]$ represents the interference channel matrix from the neighboring BS $B_{m'}$ to $D_{mk}$; here we define $\mathcal{T} = 2$ and $\mathcal{F} = 1$. It is assumed that the channel elements are independent identically distributed (i.i.d.) complex Gaussian random variables with $CN(0, 1)$ and $\mathbb{E}[s[m,k]s[m,k]H] = I_{L_s}$. The transmit precoder at $B_m$ satisfies the power constraint $\sum T[m,k]H[m,k] \leq P_m$, where $P_m$ is the maximum transmit power of $B_m$. The estimated output vector at $D_{mk}$ is obtained with the receive filter $U[m,k] \in C^{L_s \times L_s}$ as $\hat{s}[m,k] = U[m,k]H[m,k]y[m,k]$.

III. TWO-CELL ZF-IA METHOD


To achieve the asymptotically interference-free DoF (per-cell) of $\frac{K}{K+1}$ without BS cooperation, the transmit precoders are first given as $T[m,k] = PV[m,k]$ where $P \in C^{M \times K}$ is introduced for a BS to spread $K$ data streams using $M(> K)$ transmit antennas and the beam-forming vector $v[m,k] \in C^{K \times 1}$ is for cancelling IUI within the same cell.

The IA scheme of [4] assumes single stream reception with each receive filter $U[m,k]$. The receive filter output at $D_{mk}$ is given by (2), where $\bar{H}[m,k] \triangleq H[m,k]P$. To null out the ICI, $U[m,k] \in C^{L_s \times 1}$ is the null space of $\bar{H}[m,k]$. The third term on the right hand side of (2) gets cancelled out. Note that $\bar{H}[m,k]$ is assumed to be available at $D_{mk}$ via estimation. To guarantee the existence of the receive filters $\{U[m,k]\}$, the dimension of the spreading matrix $P$ must be $(K+1) \times K$. The remaining IUI is cancelled with a transmit channel inversion method, i.e., by setting $\hat{V}[m,k] \in C^{K \times 1}$ such that $U[m,k]H[m,k]V[m,i] = 0 (i \neq k)$.

B. Generalized ZF-IA

We now consider generalization to allow $L_s(> 1)$ streams at each user. First, to enable each BS to transmit $L_s$ streams to each of its $K$ users, we need to pick the spreading matrix $P$ so that the existence of the null space for $\bar{H}[m,k]$ with rank $= L_s$ is guaranteed. An arbitrary $(K+1)L_s \times KL_s$ full rank matrix whose columns are orthonormal to each other, i.e., $P^H P = I_{KL_s}$, will do the job. Now, the received signal (1) can be written as

$$y[m,k] = \bar{H}[m,k]V[m,k]s[m,k] + \bar{H}[m,k] \sum_{i \neq k}^{K} V[m,i]s[m,i] + \bar{H}[m,k] \sum_{i=1}^{K} V[m,i]s[m,i] + n[m,k]$$

where the dimension of the transmit-beam-former $V[m,k]$ is $KL_s \times L_s$.

To null out the ICI, the front portion of the receive filter $\hat{U}[m,k] \in C^{L_s \times L_s}$ is chosen from the null space of $\bar{H}[m,k]$, as can be obtained from the singular value decomposition:

$$\text{SVD}(\bar{H}[m,k]) = [U[m,k], \hat{U}[m,k]\Sigma[m,k]]V[m,k]^H$$

Now $B_m$ can perform block diagonalization (BD) to eliminate IUI. A portion of the BD precoder, $V[m,k]$, is identified from

$$\text{SVD}(H_c[m,k]) = [U_c[m,k], \Sigma_c[m,k]]V[m,k]^H$$

where $H_c[m,k] = [\Omega_{m,1}^H, \cdots, \Omega_{m,m-1}^H, \Omega_{m,m+1}^H, \cdots]H$ with $\Sigma_c[m,k] \neq U[m,k]H[m,k]$. We will write the final generalized ZF-IA transceiver solutions as $\text{GZF-IA} = P[V[m,k]H[m,k]H[m,k]P]$. The effective channel $H[m,k] \in C^{L_s \times L_s}$ and the effective noise $\hat{n}[m,k] \in C^{L_s \times 1}$ are defined as $H[m,k] = U[m,k]H[m,k]P[m,k]$ and $\hat{n}[m,k] = \bar{U}[m,k]H[m,k]P[m,k]$, respectively. The matrices $\hat{V}[m,k]$ and $\hat{U}[m,k]$ are identified by channel diagonalization:

$$\text{SVD}(H_{eff}[m,k]) = [U[m,k], \Sigma[m,k]]V[m,k]^H$$

Note that because $U[m,k]$ and $\hat{U}[m,k]$ are composed of orthonormal columns, $\mathbb{E}[(\hat{U}[m,k]H[m,k]P[m,k]V[m,k]^H)^2] = \sigma_n^2 I_{KL_s}$.

The information rate for $D_{mk}$ can now be computed as

$$R[m,k]_{ZF-IA} = \log \{ \det(I_{KL_s} + \sigma_n^{-2} \Sigma[m,k]2\Phi[m,k]) \}$$

Because this scheme causes no ICI, the sum rate over the $m$-th cell $\sum_{k=1}^{K} R[m,k]_{ZF-IA}$ is independent of the power allocation at $B_m$. Thus, the sum-rate-maximizing power allocation problem divided into the following individual-cell sum-rate-maximizing problem (in which the optimal power allocation matrix $\{\Phi[m,k]\}$ is calculated using the water-filling solution):

$$\max_{\{\Phi[m,k]\}} \sum_{k=1}^{K} R[m,k]_{ZF-IA} \quad \text{subject to} \quad \sum_{k=1}^{K} \text{Tr}(\Phi[m,k]) \leq P_m.$$ (7)

Let us call this scheme generalized ZF-IA (GZF-IA). Note that the proposed GZF-IA scheme still preserves $\frac{KL_s}{K+1}L_s = \frac{K}{2(K+1)}L_s$ DoF per cell and is implemented without BS cooperation.
\[ s[m,k] = u[m,k]^H H_m s[m,k] + u[m,k]^H H_m \sum_{i \neq k} v[m,i] s[m,i] + u[m,k]^H \sum_{i=1}^K v[i,m] s[i,m] + u[m,k]^H n[m,k] \]  

**IV. Proposed Regularized ZF-IA Method**

**A. Transceiver design**

To overcome the characteristic limitation of ZF schemes - namely, the sum-rate performance suffers in the low-to-mid SNR regime - we devise a regularized GZF-IA algorithm based on the well-known notion of regularized channel inversion [8]. Regularizing the channel inverse leads to minimization of the weighted MSE defined as

\[
\min_{m=1}^{2} \sum_{k=1}^{K} \mathbb{E}[|\Lambda[m,k] s[m,k] - \hat{s}[m,k]|^2]
\]

subject to \(\sum_{k=1}^{K} \text{Tr}(T[m,k] T[m,k]^H) \leq P_m, \forall m\)  

where \(\Lambda[m,k]\) is introduced to improve the sum-rate performance by preventing the assignment of higher power to weaker subchannels [8]. Accordingly, \(\Lambda[m,k]\) is chosen as the effective channel gain matrix to \(D_{mk}: \Lambda[m,k] = \frac{u[m,k]^H H_m}{\mu_m^2} T[m,k] u[m,k]^H\). Then the Lagrangian function associated with (8) can be written as (9), where \(\{\mu_m\}\) is the Lagrangian multiplier. Because the transceiver matrix \(\{V[m,k]\}\) and \(\{U[m,k]\}\) are inter-related, we resort to alternating optimization that would lead to local optimum.

First, we design the optimal precoder assuming the receive filters are given. From \(\nabla V[m,k], \mathcal{L} = 0\), the precoder for \(D_{mk}\) is obtained as:

\[
V[m,k] = \left( \sum_{i=1}^{K} \sum_{i=1}^{K} \mathbb{E}[n_i^2] + \mu_m^2 I_{K_L} \right)^{-1} H_m^H U[m,k] \Lambda[m,k] \]

(10)

where \(\mathbb{E}[n_i^2] = H_m^H u[m,i]^H u[m,i] H_m\) with the initial filter setting \(\{U[n_i]\}\) naturally taken from the GZF-IA scheme of the last section. Since the \(m\)-th BS transmit power is a monotonically decreasing function with respect to \(\mu_m\) (the proof is omitted due to the space limitation), the value of \(\mu_m\) that satisfies the power constraint can be found efficiently using a bisection method.

Next, we derive the receive filters \(\{U[m,k]\}\) based on the precoders \(\{V[m,k]\}\) obtained in (10). The optimal receive filter for \(D_{mk}\) is derived with \(\nabla U[m,k], \mathcal{L} = 0\) and is shown to be

\[
U[m,k] = \left( \sum_{i=1}^{K} \sum_{i=1}^{K} \Psi[i,n_i] + \sigma_m^2 I_M \right)^{-1} H_m^H V[m,k] \Lambda[m,k]^H \]

(11)

where \(\Psi[i,n_i] = H_m^H v[i,n_i] v[i,n_i]^H H_m\).

In summary, the overall algorithm is based on initially starting out with the receive (or transmit) filters of the GZF-IA scheme and repeatedly updating the transceiver filters by alternately utilizing (10) and (11). This algorithm is provably convergent at least to a local minimum.
\[ \mathcal{L} = \sum_{m=1}^{K} \sum_{k=1}^{\frac{H}{2}} \text{Tr} \left\{ (\mathbf{A}^{[m,k]} \mathbf{U}^{[m,k]} \mathbf{H}^{m,k} \mathbf{V}^{[m,k]} \mathbf{A}^{[m,k]} \mathbf{H}^{m,k}) \mathbf{U}^{[m,k]} + \sigma_n^2 \mathbf{U}^{[m,k]} \mathbf{H}^{m,k} \mathbf{U}^{[m,k]} \right\} + \sum_{m=1}^{K} \sum_{i=1}^{\frac{M}{2}} \mu_m \left( \sum_{k=1}^K \text{Tr} \left( \mathbf{V}^{[m,k]} \mathbf{V}^{[m,k]} \mathbf{H}^{m,k} \right) - P_m \right) \]  

(9)

Fig. 2. Sum-rate convergence behaviors of RZF-IA vs. max-WSR

Fig. 3. Sum-rate performances at different numbers of iterations.

VI. NUMERICAL RESULTS

For sum-rate performance simulation results, we set \( M = 6, K = 2, L_u = 2, P_1 = P_2 = P \). The SNR is defined as \( \frac{L_u}{\sigma_n^2} \). Fig. 2 illustrates the convergence behavior of the RZF-IA method (thick lines) and the max-WSR method (thin lines). As the converged performance of the max-WSR schemes depend on the initial precoder setting in the alternate transceiver optimization process, an average is taken over a large number of different random initial settings (more than 50) for a given channel realization. For both schemes, the performance curves reflect averaging over a large number of independent channel realizations. The plots of Fig. 2 reflect the averaged performance. The plots show that while RZF-IA is not as good as max-WSR as a large number of iterations is allowed at high SNRs, the former algorithm converges faster than the latter method. In fact, at a reasonably small number of iteration, RZF-IA performs better than max-WSR on average.

Fig. 3 shows the sum-rate performance at different numbers of iterations \( I_1 = 1, 2, 100 \), again averaged over many channel matrix realizations as well as initial precoder settings in the case of max-WSR. Specifically, at \( I_1 = 2 \), due to the fast convergence, RZF-IA indeed shows better performance than max-WSR. We also confirm that RZF-IA enhances the performance of GZF-IA. At a sufficient number of iterations, e.g., at \( I_1 = 100 \), and at high SNRs, the RZF-IA scheme shows a noticeable performance degradation compared to max-WSR due to the sub-optimality of the MSE weights, a price paid for reduced computational complexity and required information exchange.

VII. CONCLUSION

We extended the ZF-IA method of [4] to the two-cell MIMO IBC where each user receives multiple data streams. We subsequently proposed a regularized ZF-IA method to improve its sum-rate performance. To this end, a minimum WMSE method has been applied with the weights computed from the effective channel gain of the initially constructed generalized ZF-IA scheme. WMSE minimization is achieved via alternating calculation of the linear transmit and receive filters. Since the weights used in the WMSE minimization comes directly from the generalized ZF-IA solution, the proposed scheme requires less information exchange overhead and converges faster to the full performance level compared to the existing max-WSR methods that require alternating iterative computations of the weights. Through analysis and simulation, the effectiveness of the regularized ZF-IA scheme has been validated.

REFERENCES


